Dummett's Anti-Realism about Mathematical Statements

Jan Štěpánek

Filozofická fakulta
Masarykova univerzita
Arna Nováka 1, 602 00 Brno
380011@mail.muni.cz

Just as the accuracy of scientific theories is best tested in extreme physical conditions, it is advisable to verify the accuracy of a recognized conception of language on its extreme parts. Mathematical statements meet this role, thanks to the notion of truth and proof. Michael Dummett's anti-realism is an enterprise that has attempted on this basis to question the notion of the functioning of language-based primarily on the principle of bivalence, the truth-condition theory of meaning, and the notion that the speaker must be able to demonstrate his knowledge of meaning publicly. In common language practice, one can observe assertions that we can neither verify nor refute in principle. On these so-called undecidable statements, Dummett tried to show that if we apply the traditional description to them, we inevitably reach paradoxical conclusions. Mathematical statements referring to an infinite number may be examples of these assertions. In the submitted paper, I will present Dummett's position resulting primarily in a manifestation and acquisition argument, according to which it should not be possible to understand undecidable statements at all. In conclusion, however, I will show that his intention – despite many valuable comments – fails, i.e. that there is a way to avoid both arguments while preserving the realistic description of the language in general.

Key words: anti-realism, mathematical statements, meaning, Michael Dummett, truth, truth-condition theory of meaning

At first glance, mathematical statements are no different from ordinary assertions. Although they contain some symbolism, we use everyday expressions to express them, translate them into foreign languages, and determine whether they are true or false. In many other ways, however, they are an exceptional part of the language. The semantic differential of mathematical theorems is close to zero, meaning is given once and for all, and proof plays a key role alongside the truth.[1] Unlike empirical sentences, it is possible to doubt what mathematical statements refer to, i.e. what the ontological status of mathematical objects is.[2] There can be no doubt, on the other hand, that the accepted theory of meaning must be sufficiently precise and general to describe all parts of the language uniformly. It is just the mathematical statements that, by virtue of their properties, will serve as a suitable adept for examining the correctness of these theories.

The same intention in the philosophy of language was started by the British philosopher Michael Dummett, who used the basic principles of intuitionism to challenge the dominant description of language at that time.[3] In the present article, I will attempt to interpret and critically evaluate his position, using the example of mathematical statements. For, while Dummett's criticism seems to be appropriate, one may doubt his description of a realistic position as I will call the dominant theory of language. I even believe we can avoid his arguments even at the cost of accepting all the underlying assumptions. But first, in the following article, I will briefly present the notion against which Dummett defined himself. Then I will show his criticism, and briefly also the theory of anti-realism built on it, which seeks the solution of defects. In conclusion, I will use a few examples to assess the strength of the objections by which Dummett competed with the rival notion, and to show how a realist can defend himself.[4]

1 When we talk about proof, we need to remember whether we interpret it from the perspective of classical or intuitionistic mathematics and logic.

2 Among others, this question motivated the emergence of several opposing approaches to mathematics. The most famous of these were formalism, logicism, and constructivism, a special case of which is intuitionism, which was in many ways a source of inspiration for Dummett. See Dummett (1977, p. 1).

3 For first texts of this kind see Dummett (1996a) and Dummett (1996b).

4 Although the theory of meaning must be universal in its description of the language, this does

The classical conception of language

For our purposes, it is not necessary to work with a precise definition of a mathematical statement. It is quite enough to understand such a sentence that works with natural numbers. For example, we can take three well-known sentences, the one about the existence of infinitely many primes, the one about doubling the cube, and the Goldbach's conjecture, which claim in turn:

(E) There are infinitely many primes.
(Z) It can be created a cube whose volume is doubled that of a given cube by using a ruler and compass.
(G) Any even number greater than two can be written as the sum of two primes.

The difference between them is that in the case of (E) and (Z) we have proofs that confirm the first sentence and refute the second. For (G), however, despite our best efforts, we do not yet have proof.

According to realistic semantics, all mathematical sentences – regardless of the truth or presence of proof – speak of mathematical objects. Primes, cubes, etc. do not have their existence depending on thinking beings but live their lives independently of us. If not in ordinary space-time, then at least in some realm of abstract entities. All claims about them are either true or false once and for all. There is no third value. Even (G) or statements that have not yet been formulated already have their truth value. We just do not yet know what it is. For this reason, this concept of truth is called verification transcendent. For even such claims, the difficulty of which exceeds our confirmatory capabilities currently or in principle, are true.[5]

not mean that it is necessary to hold the same position in all different discourses. One can be a realist about theorems of past tensed sentences and at the same time an anti-realist about mathematical statements. See Dummett (1996d, p. 374).

5 Of course, at this point, we do not know whether Goldbach's conjecture is undecidable in principle. At any time, some evidence may emerge. But at the moment, we do not know any procedure for how any even number greater than 2 could be expressed as the sum of two primes. Dummett's original example of an undecidable infinite statement is the sentence "A city will never be built on this spot." Dummett (1996a, p. 16). But I do not think it is appropriate because, although confirmation of the statement requires an "infinitely" long time, a situation may occur without difficulty in real-time to disprove it.

The meaning of the above sentences is – I suppose – obvious and, just by looking at Goldbach's conjecture, clearly has nothing to do with the knowledge of the relevant proof. Although it is we who formulate these sentences, it seems that each statement has meaning without our doing, that the statements are connected to the world and, through it, to a particular truth value. Meaning and truth are thus two sides of the same coin. Sentence (E) expresses the fact that there is no greatest prime number. And because we know through the proof that this is indeed the case, we consider (E) to be true. If a sentence expressed such properties that primes do not have, it would be false. Knowledge of meaning is thus an important component for us to identify the truth value.[6]

The connection of meaning and the concept of truth is a key part of the truth-conditional theory of meaning, a theory that plays a central role in the philosophy of language and which we will associate with a realistic view of language. Its proponents insist that the meaning of a sentence is given by its use and knowing it means to be able to use the statement unproblematically or to know the conditions under which the sentence is true. To know the meaning of the sentence "Prague is the capital of the Czech Republic" means to know how the world must be set up for the sentence to be true. For Prague to be the capital of the Czech Republic, it needs to have a parliament, a president, and for this information to be marked in maps and atlases. But what would the world have to look like for the theorems of mathematics to be true? Proponents of the idea of the existence of mathematical reality would say that the guarantee of the truth of the theorems is existing objects with described properties.[7] But someone else might argue that there must be proof. And it does not matter whether we know it. But it is enough to know that a deductive correct procedure exists and is traceable in the scientific literature.

Another important aspect of a realistic description of a language is that sentences do not have meanings of their own, but that they form a huge network in which they are interconnected. The meaning of one sentence thus depends on the meanings of many other sentences, and to fully understand it, the whole network must be understood. The propo-

6 Tautological sentences are exceptions, e.g. $\neg\neg A \leftrightarrow A$.

7 Dummett (1993d, p. 248).

sition (Z) and the sentences “The volume of a cube is calculated as the third power of the length of the edge” and “A cube is a convex body” not only do not contradict each other, but the meaning of one sentence even completes the meaning of the others.

In summary, we can say that the basic features of a realistic description of a language are the above-mentioned principle of bivalence, truth-condition theory of meaning, the verification-transcendent concept of truth, and a holistic approach to a language. Any sentence in our language should therefore be capable of being described using these tools, and its meaning should not contradict our intuitions. This means that there cannot be a situation where we find a sentence meaning of which we clearly understand, but the realistic description of which does not allow us to understand it. If this were the case, the theory would have to be rearranged. It was this idea that drove Dummett in his verification of the correctness of a realistic conception of a language.

Objections to the truth-condition theory

Before explaining Dummett’s example and arguments, we will take a closer look at what else acceptance of the theory obliges us to do. The ability to imagine truth conditions depends on the ability to master a language, understand it, and recognize which sentences make sense and which do not. Part of this background, according to Dummett, is the theory of meaning.[8] This can be seen, for example, from the fact that if we could program a language into a robot to use it and understand it, then we could not do without the theory of meaning. Without its knowledge, a talking robot would not be able to work independently with the truth of even elementary sentences. Therefore, the practical ability to use a language requires a theory of meaning, i.e. a set of information – or rules – on what sentences mean and how they relate to the world.

But how are we familiar with the theory of meaning, or of the truth? It can be either a set of explicitly expressed sentences, or a set of internal implicit habits. The first possibility is contradicted by the fact that, although most speakers of a language can communicate without much

8 Dummett (1993b, p. 36–37).

difficulty, only a very small percentage of them can formulate, for example, the Tarski T-scheme. And even if someone can do so, their efforts to express their understanding of a sentence will always end up just by saying another sentence. It makes no sense to say that I understand the sentence (Z) because I also understand the statement "The cube is a specific case of a cuboid." I run the risk of being asked to show how I understand this new sentence. So I would keep creating more and more sentences, which would put me either in a *circulus vitiosus* or in an infinite regress.[9]

The conclusion is that if knowledge of meanings cannot be explicit, it must be implicit. Let us assume that there are sentences that the speaker understands internally, intuitively, without the ability to say exactly how. The idea that inexpressible knowledge would lie in the speaker's private states, and that it would make it possible to communicate with the language as a public medium without difficulty at the same time, does not make sense, according to Dummett.[10] It would be a problem like that of the existence of a private language. The only way to maintain a theory of meaning based on truth conditions is to resort to the view that the knowledge of meaning is manifested in the language behaviour of the speaker. That is a faculty that manifests itself publicly and can be observed and understood as an act of understanding by anyone who speaks the same language. The role of implicit knowledge can be played, for example, by nodding one's head in agreement in a situation where a sentence-expressed event occurs, or by another rational reaction that does not contradict the sentence uttered and on the contrary complements it. The important point is that the meaning must be completely publicly accessible. If any part of it were private and inexpressible, then no one would be able to reach that part of the meaning, making it completely useless for any communication.[11]

Admittedly, it takes a lot of situations, a lot of agreeing and no less disagreeing to test a single sentence. Moreover, we go through the same test every time we debate. Even when the computer bots check that we are real people and let us choose the parts with the traffic light from

9 Dummett (1996c, p. 217).

10 Ibid, p. 217–218.

11 Dummett (1977, p. 260).

a picture of a busy city street. In this way, they verify that we understand the sentence "Here is the traffic light" and that we are humans. But how do we prove our understanding of the sentence about the existence of infinitely many primes? What must the world look like for (E) to be true? It is fair to say that in that abstract realm of mathematical objects there must be an infinite number of natural numbers that have exactly two divisors. Confirmation may be the existence of formal proof, and in another way, the existence of the so-called Mersenne primes, a procedure that allows us to construct freely sized primes with no limitations.

Thus, even the knowledge of the method of confirming or disproving the statement is sufficient to prove our understanding of the meaning. However, the condition is that this procedure can be performed in a finite time and a finite number of steps. One non-mathematical example is the sentence "50 miles below the surface of the Martian equator there is water."[12] While we cannot currently confirm or refute this sentence, we can imagine what the world is supposed to look like for it to be true, and we can even imagine a way to decide it. It is sufficient to send a crew to the Red Planet. However, there is no way we can reverse the passage of time to demonstrate an understanding of the subjunctive conditional "If Clinton had been born in New York, he would never have become president." Nor can we postulate a procedure to at least hypothetically verify, or refute this sentence. There is no method by which we can manifest our implicit knowledge, and we can only conclude that it is impossible to understand this sentence according to the assumptions above.[13] Certainly not based on the principles that realists accept. The theory of meaning based on truth conditions has led us to a non-intuitive conclusion and we should therefore abandon it. In the case of mathematical statements, the situation is analogous. (G) is a typical example.[14]

Given the emphasis that has been put on demonstrating the ability to understand a sentence, the previous argument is referred to as a mani-

12 Loux (2003, p. 640).

13 Dummett (1996c, p. 224–225).

14 At this point, the terminology should be clarified. Dummett's term "undecidable" must be understood in the context of truth condition theory. States of affairs of these statements are beyond our verification abilities. On the other hand, the term "undecidable" has a different meaning in mathematics, where it refers to a statement that is proven to be impossible to prove within a given theory. Just as the continuum hypothesis within the ZFC.

festation argument, and in this article, we will – in line with Alexander Miller[15] – distinguish between a strong and a weak version. The structure of the two arguments can be described using the following statements:

(1) The understanding of sentences depends on the knowledge of their truthfulness conditions.
(2) The truthfulness value of sentences is verifiably transcendent.
(3) The understanding of sentences is a set of practical skills to use them.

The judgment itself takes then the form of:

$$\frac{((1) \wedge (2)) \Rightarrow \neg (3) \qquad (3)}{\neg ((1) \wedge (2))}$$

On the example of undecidable statements, truth conditions of which we cannot encounter in any way, nor can we possess a procedure to give a sentence truth value, we can see that all three statements cannot be made at the same time. Either (1) and (2) must be valid, or (3). Dummett is inclined to think that it is necessary to reject (1), i.e. the truth condition theory of meaning, and replace it with a more accurate description. According to the strong version of the manifestation argument, it is, therefore, necessary to reject realistic semantics.[16] The weak manifestation argument differs from the strong one in the final opinion. Its position is that a realistic concept must be complemented by something, or that it cannot be accepted based solely on the practical ability to show an understanding of a sentence.[17]

The acquisition argument, according to which it is even impossible to learn the meanings of undecidable sentences, is closely related to the manifestation argument. To learn the meaning of sentences, it is not

15 Miller (2002, p. 354–355).
16 Ibid, p. 356.
17 Ibid, p. 360.

necessary to have precise definitions of all the expressions involved. It is rather a pragmatic matter, the key component of which is the art of controlling sentences in practice, using them in convenient situations, and seeing an adequate response in the surroundings. This is the way by which we learn the meanings. We learn the meanings of sentences describing facts within our reach by observing other speakers as they repeatedly use them in the presence of the same events. By combining observation, trial, and error, we consolidate, modify, or completely reject the meaning of a sentence if it conflicts with other sentences. I learned the meaning of the statement "Number 59 is a prime number" by observing other sentences about primes, divisors, etc. But the question is, how can one learn to use and understand undecidable sentences when one could never witness the realization of their truth conditions?[18]

To formally construct an acquisition argument, it is sufficient to replace (3) with an amended statement:

> (3') The meanings of sentences are acquired by observing their use.

The form of judgment is the same as that of a manifestation argument:

$$\frac{((1) \wedge (2)) \Rightarrow \neg (3') \qquad (3')}{\neg ((1) \wedge (2))}$$ [19]

Again, the crucial point of judgment is the impossibility of encountering the contents of Goldbach's conjecture. Realistic semantics force us to conclude that we do not really understand the conjecture at all, because we could never have witnessed the relevant truth conditions occurring.[20] But any speaker of a language can easily form an idea of what (G) is saying.

18 Dummett (1996d, p. 362).

19 Miller writes this argument somewhat differently. In the acquisition argument, he merges the statements (1) and (2) into a single statement. Miller (2003, p. 3). However, this does not change its validity.

20 Indeed, due to the absence of proof, Goldbach's conjecture is currently neither true nor untrue from Dummett's point of view. It gains its truthful value only when we prove it or its negation. Dummett (1977, p. 267).

The last lesser-known argument presented here attempts to disrupt the key role of the notion of truth. Truth and meaning should form an inseparable pair, at least in the sense that if we want to determine the truth value of a statement, if it is not tautology in its form, we must first know its meaning. At the same time, as we learn a language, we train ourselves to connect the states of things with individual meanings, thereby acquiring the sentence for further use. Although the truth of sentences does not depend on us – at least from a realistic conceptual point of view – we are constantly trying to discover it. In search of mathematical proof, in answering questions in a knowledge quiz, etc.

Consider the following thought experiment: At one moment, two changes occur in the minds of language users around the world. The first concerns our terminology, causing the numbers 2, 4, 6 … to be referred to as "odd" and the numbers 1, 3, 5 … to be referred to as "even". The second results in a change in attributing to the very property "to be even" and "to be odd". Thus, we would now consider 2, 4, 6 … to be odd numbers and *vice versa*. But the change would occur only in ourselves, only in our perception. The parity of the numbers themselves remains the same. Dummett then asks what the world would look loike after these changes.[21] Would it change at all? Both changes would obviously be cancelled out because the statement "The number 2022 is even" would still be accepted by every linguistically equipped person. We would continue to be able to use all known terms well enough, and we would never get into any conflict caused by the changes described above. Even Goldbach's conjecture would still have the same form.

Yet one fundamental change has occurred. Many of our statements would now be false. Even if everyone agrees with the statement about the even parity of the number 2022, the contents of this statement are that 2022 is not divisible by 2 without remainder. The old terminology called this number odd. Therefore, any connection with the actual state of affairs has disappeared, although there is nothing to prevent successful communication. Our statements about even and odd numbers do not refer to the numbers that we believe to refer to. But this means that the

21 Dummett's example did not work with mathematical concepts but with colours. However, the form of the argument is the same. Dummett (1991, p. 313–314).

relationship between truth and meaning is completely irrelevant and that a realistic concept does not constitute an appropriate description of language.

Although realists and anti-realists approach many theorems, in the same way, i.e. theorems verification or denial of which are either within our direct or at least possible reach, the two directions differ in the interpretation of undecidable statements. The inability of meaning theory, based on truthful conditions, to describe the understanding and learning of such theorems, and the related conception of truth, leads Dummett to reject a realistic position and seek an alternative. It is important to realize what Dummett is really opposing. His aim is not to reject the idea that meaning is a use. Instead, he wants to preserve this conception. We should only revise the emphasis we place on the notion of truth.[22] Dummett builds an alternative on the basic idea of intuitionism, according to which the concept of verification transcendent truth is replaced by the ability to recognize evidence or evidence, while the phrase "meaning is a use" remains valid.[23]

Dummett's anti-realism

The change that needs to be made so that we are not forced to accept non-intuitive conclusions is obvious:

> "We must, therefore, replace the notion of truth, as the central notion of the theory of meaning for mathematical statements, by the notion of proof: a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us."[24]

22 Dummett (1996c, p. 224).

23 The roots of this concept lie in Brouwer's conception of mathematics, from which the whole debate originated. Intuitionists believe that mathematics is a mental activity that is created within us. Therefore, there is no place in this theory for a static and eternal concept of truth. Ibid., p. 225.

24 Ibid., p. 225. Stuart Brock and Edwin Mares interpret Dummett's words to mean that the meaning of a statement is a set of conditions for how to verify a sentence and that it is, therefore, possible to escalate the understanding of the meaning with an increasing number of known conditions. See Brock & Mares (2006, p. 80–82). Given the inconvenience to which such a concept leads, and given Dummett's own words, we will adhere to a non-set concept of meaning. The same view can be found elsewhere, see Loux (2003, p. 641–645).

In the words of Bernard Weiss, the truth condition theory of meaning, which cannot explain how to manifest an understanding of the meaning of undecidable statements, is replaced by the proof condition account of meaning, according to which to understand the meaning of a sentence is to understand what parameters a construction must have to be a proof of a sentence. It is the ability by which we can decide on processes that are proof and which are not.[25]

Because of this concept, it is sufficient for the speaker to have the ability to recognize what the world must look like for a sentence to be true. The description of understanding the meanings (E), (Z), and (G) then produces no questionable conclusions. But Dummett's anti-realism should not be understood as requiring a language user to show an understanding of the meaning of a sentence by performing the proof of the sentence himself, or by being able to identify it among the several procedures offered. To understand the proof of a mathematical statement, it is necessary to be familiar with certain theoretical foundations, such as axioms, other usable sentences, symbolism, and, above all, how formal evidence is formed. It takes much less to understand a statement. We know its meaning if we acquire the ability to understand any proof. Dummett's theory of meaning[26] which includes all claims can also be formulated in such a way that to know the meaning of a claim is to know the circumstances in which we would be convincingly entitled to make a claim.

Weak points of Dummett's arguments

As we have seen above, an undecidable statement can also be a theorem that speaks of an infinite set of numbers and for which we do not know a method that can be done in a finite time to confirm or disprove the statement. One example is the Goldbach conjecture (G). The phrase "Number 987655111 is a prime number" is, on the other hand, determinable because we know an algorithm – the Eratosthenes sieve

25 Weiss (2002, p. 98).

26 For a long time, Dummett referred to his theory of meaning as verificationist, though it differed substantially from the positivist theory of the same name. To avoid misunderstanding he later began to work with the concept of justification. Dummett (1993e, p. 475).

– that can reliably confirm or disprove it. But sentences like (G) are the reason why Dummett finds truth-condition theory inappropriate. We have raised 3 objections to its proponents in the previous text. The manifestation argument in its strong and weak form, the acquisition argument, and the questioning of the key role of the notion of truth. In this section, I will show that each of these objections can be avoided while retaining the essential elements of a realistic description of language.[27]

A defense against a strong version of a manifestation argument is a position that can ensure the validity of (1), (2), and (3) simultaneously. How this can be achieved requires finding the ability by which a speaker can demonstrate his understanding of undecidable sentences. Miller shows in a discussion between proponents of both sides that this skill can be the ability to decide whether a sentence is describable by truth conditions or not.[28] A speaker of a language in manifesting an understanding of the meaning of (G) does not need to provide any proof, nor does it need to – and indeed cannot – refer to the truth conditions of a previously unproven conjecture. It is enough to show that knowledge that there is no proof for (G) yet, or that it is not possible to present truth terms for a given sentence, i.e. that a sentence is undecidable with Dummett's terminology. A strong version of the argument is therefore not acceptable and cannot be understood as a reason to directly deny a realistic concept.[29]

In McDowell's note, Miller goes on to show that a manifestation

27 However, there are many more problems surrounding the intuitionist conception of mathematics and anti-realism. We will focus here only on the outright objections to the realistic semantics of theorems, but anti-realists can be accused of, among other things, certain carelessness in the use of the concepts of negation and undecidability, which need to be interpreted from the point of view of intuitionist logic and thus consistently distinguished from the concept of classical logic. For this see Diez (2003). From another point of view, James Page also defined himself against Dummett-who, although he also focused on the problem of the manifestation argument, dealt with specific terms, not whole sentences. See Page (1991).

28 Miller (2002, p. 360).

29 A similar solution was proposed by Geoffrey Hunter (1980, p. 122–123). According to him, an understanding of the meaning of (G) can be demonstrated by being able to select from a list of those theorems that are equivalent to it, i.e. that express the same content. If the theorems (G1) stand before us, "For every even natural number greater than 2, one can find a pair of numbers that add up to the original number and have exactly two divisors for each." and (G2) "All natural numbers greater than 2 can be written as the sum of two primes." then if the speaker identifies only (G1) as equivalent to (G), we can readily understand its selection as a proof of the theorem's proposition.

argument cannot be used to prove that realistic semantics need some correction. He shows that truth conditions theory correctly describes undecidable sentences. Knowledge of sentence's truth conditions is not conditioned by the ability to be a direct witness to them. The phrase "to understand the meaning of a sentence is to know or to understand its truth conditions" obliges us only to be able to say what events would confirm or refute the sentence. Even without knowledge of the procedure to prove truth (G), we can understand the conjecture precisely because we know what such construction should do.[30]

Unlike many other thinkers, Dummett does not accept the holistic concept of language. "[T]he acceptance of holism should lead to the conclusion that any systematic theory of meaning is impossible … my own preference is, therefore, to assume as a methodological principle that holism is false."[31] Each sentence has a meaning by itself. If the meaning of sentences depends on the meaning of other sentences, it would be impossible to learn any language at all. To master a language would require knowledge of all the propositions and the rules associated with them. Dummett on the other hand accepts the molecularity of meaning theory that understanding any sentence "will depend upon the mastery of some fragment of the language, more or less extensive according to the complexity or depth of the sentence."[32] The meaning of a statement is made up of the meanings of its parts. Thus, it seems possible to understand sentences like Goldbach's conjecture simply because we know the meaning of the sentence "Every even number greater than 2 and less than 100 is the sum of two prime numbers," which is decidable and therefore also easily manifested, and we also understand the concept of infinity[33], which is just the constant addition of another successor. By unifying these known truth conditions, we can then learn the meaning of the proposition (G), which at first glance may seem too complicated.

30 Miller (2002, p. 367).

31 Dummett (1993a, p. 21).

32 Dummett (1977, p. 255).

33 Dummett seems to argue that from a realism point of view, no statement about infinite quantities is comprehensible because it sets us the task of holding an infinite number of elements at one moment. But this objection can be easily dismissed by reference to the axiom of infinity set theory, the function of the successor in Peano arithmetic, etc. It is sufficient to work with infinity potential instead of infinitely current.

The compositional principle is a powerful tool to which anti-realists have no effective answer. In the case of the acquisition argument, it allows the proposition (1), (2) and (3‘) to be held simultaneously because we can also reach an understanding of sentences by composing previously acquired meanings and knowing their truth conditions. We also learn other propositions by composing meanings. “50 miles below the Martian equator is water” is just a combination of the terrestrial form of this theorem and the planet Mars. The phrase “Hannibal’s favourite geometric shape was a triangle” is again a mere combination of simpler meanings and therefore simpler truth conditions. As proof, one can even cite a sentence that none of us have ever heard before, i.e. could never even face its truthful conditions in its entirety, and yet it will be understood immediately by “Gandalf the Grey entered McDonald’s and ordered sushi.” The compositional principle is therefore an effective response to the acquisition argument.[34]

Andrew Ward made another criticism.[35] According to him, Dummett omits the difference between verifying the truth of a statement and the justification to attribute truth conditions to a sentence.[36] The first case concerns the correspondence between the meaning of a sentence and reality. The second then – related to the principle of coherence – the relation between each sentence. Therefore, for Dummett, the only way to prove knowledge of the meaning of a sentence, truth conditions of which we are unable to verify at present, is to find a procedure that would at least hypothetically put us in this role. However, if we consider the difference between the two situations, it is entirely legitimate to manifest an understanding of a sentence also by means of sentences directly related to it, or by explaining “the role (semantic location) of the sentence in the logical space provided by the other sentences of the language.”[37]

34 Miller (2003, p. 469–470).

35 Ward (1988, p. 554–555).

36 This can be seen, for example, in Dummett reading realism. To maintain the principle of bivalence while avoiding a vicious circle, he submits to realists an attitude according to which there must be something in the world that causes the truth or falsity of claims. Dummett (1996a, p. 14).

37 Ward (1988, p. 555).

The connection of individual sentences and concepts are crucial points for the last objection. The thought experiment of confusing the terms "even" and "odd" and confusing our perceptions of parity seems convincing. Humans go through a psychological change, but mathematical objects do not. This, according to Dummett, causes a paradoxical situation: Compared to mathematical reality, our statements are now false, but they also serve to communicate successfully. What the experiment neglects is that the predicates "even" and "odd" do not stand alone in our mathematical theories but are linked to many other concepts. If there is to be confusion at the level of predicates and not just our expressions, the related concepts must be changed along with the predicates. "Being even" carries the predicate "being divisible by two without remainder." But if, after the change, we mark the numbers 1, 3, 5 … as even, i.e. numbers divisible by two without remainder, then we can clearly see that this condition is not fulfilled. So, we would have to modify the division operation as well. But such a change already leads to further adjustments, and if they were done with real precision, we would find that we need to transform the plethora of expressions in which we talk about mathematical objects. At the end of this effort, however, we would get to a state where our completely rebuilt statements still accurately describe numbers and retain their truth as before the initial change. The only difference would be a change in terminology.

Conclusion

Dummett's theory of anti-realism can be criticized from almost all points of view. We have concentrated exclusively on the problems of his objections to realistic semantics. But for its beauty and pertinence, let us also cite an objection concerning Dummett's division into sentences that are decisive and undecidable. John McDowell agrees to some extent with Dummett's criticism of realistic concepts.[38] But he does not see from which position we can say about some assertions that we can never decide them, namely that their truth conditions are beyond our verification abilities.

38 McDowell (1989, p. 180).

> "Dummett's realist standardly aims to credit himself with a conception of what it would be for something to be capable in a way that transcends his own capacities to tell it is true, by exploiting the notion of a notion of whether or not cognitive cities are being limited – a whole of a direct capability into the whole of some pictured region of reality."[39]

Over the years, Dummett has revised some of his views in response to criticism. He has acknowledged that mastery of a language is not purely practical art or skill, but that it requires a theoretical basis in addition to practice, and that it is a phenomenon so complex that it is very complicated to describe its functioning in detail. Nevertheless, he does not give up on the importance of practical knowledge.[40] As I have attempted to show, the arguments he has made against a realistic view of language are not convincing. Certainly not in the form in which he has formulated them. Yet his ingenious criticism has revealed several issues, and the debate he has provoked has shown the way forward.

References

Brock, S. & Mares, E. (2006): *Realism and Anti-Realism*. Routledge, London.

Devitt, M. (1983): "Dummett's Anti-Realism." *The Journal of Philosophy* 80 (2): 73–99.

Diez, F. (2003): "Is the Language of Intuitionistic Mathematics Adequate for Intuitionistic Purposes?" *Logic and Philosophy of Science* 1 (1).

Dummett, M. (1977): *Elements of Intuitionism*. Oxford University Press, Oxford.

Dummett, M. (1991): "Truth-Conditional Theory of Meaning." In *The Logical Basis of Metaphysics*, M. Dummett, Harvard University Press, Cambridge, 1991, p. 301–321.

Dummett, M. (1993a): "What is the Theory of Meaning? (I)" In *The Seas of Language*, M. Dummett, Oxford University Press, Oxford, 1993, p. 1–33.

39 Ibid., p. 185.

40 Dummett (1993, Preface).

Dummett, M. (1993b): "What is the Theory of Meaning? (II)" In *The Seas of Language*, M. Dummett, Oxford University Press, Oxford, 1993, p. 34–93.

Dummett, M. (1993c): "What is Mathematics About?" In *The Seas of Language*, M. Dummett, Oxford University Press, Oxford, 1993, p. 145–165.

Dummett, M. (1993d): "Realism." In *The Seas of Language*, M. Dummett, Oxford University Press, Oxford, 1993, p. 230–276.

Dummett, M. (1993e): "Realism and Anti-Realism." In *The Seas of Language*, M. Dummett, Oxford University Press, Oxford, 1993, p. 462–478.

Dummett, M. (1996a): "Truth." In *Truth and Other Enigmas*, M. Dummett, Harvard University Press, Cambridge, 1996, p. 1–24.

Dummett, M. (1996b): "Realism." In *Truth and Other Enigmas*, M. Dummett, Harvard University Press, Cambridge, 1996, p. 145–165.

Dummett, M. (1996c): "The Philosophical Basis of Intuitionistic Logic." In *Truth and Other Enigmas*, M. Dummett, Harvard University Press, Cambridge, 1996, p. 215–247.

Dummett, M. (1996d): "The Reality of the Past." In *Truth and Other Enigmas*, M. Dummett, Harvard University Press, Cambridge, 1996, p. 358–374.

Hunter, G. (1980): "Dummett's Arguments about the Natural Numbers." *Proceedings of the Aristotelian Society New Series* 80: 115–126.

Loux, M. J. (2003): "Realism and Anti-Realism: Dummett's Challenge." In *The Oxford Handbook of Metaphysics*, eds. M. J. Loux & D. W. Zimmerman, Oxford University Press, Oxford, 2003, p. 633–664.

McDowell, J. (1989): "Mathematical Platonism and Dummettian Anti-Realism." *Dialectica* 43 (1): 173–192.

Miller, A. (2002): "What is the Manifestation Argument." *Pacific Philosophical Quarterly* 83 (4): 352–383.

Miller, A. (2003): "What is the Acquisition Argument." In *Epistemology of Language*, eds. A. Barber, Oxford University Press, Oxford, 2003, p. 459–497.

Page, J. (1991): "Dummett's Mathematical Antirealism." *Philosophical Studies* 63 (3): 327–342.

Ward, A. (1988): "A 'Semantic Realist' Response to Dummett's Antirealism." *Philosophy and Phenomenological Research* 48 (3): 553–555.
Weiss, B. (2002): *Michael Dummett*. Routledge, London.

Abstrakt

Dummettův anti-realismus o matematických tvrzeních
Podobně jako se správnost vědeckých teorií nejlépe testuje v extrémních fyzikálních podmínkách, je vhodné ověřovat správnost uznávaného pojetí jazyka na jeho extrémních částech. Vhodným adeptem na tuto roli jsou díky pojmu pravdy a důkazu matematická tvrzení. Anti-realismus Michaela Dummetta je podnik, který se právě na tomto základě pokusil nabourat dosavadní představu o fungování jazyka založeného především na principu bivalence, teorii pravdivostních podmínek a představě, že mluvčí musí být schopen svou znalost významu veřejně vykázat. V běžné jazykové praxi lze zpozorovat tvrzení, která nedokážeme v principu ani potvrdit, ani vyvrátit. Na těchto tzv. nerozhodnutelných tvrzeních se Dummett pokusil ukázat, že pokud na ně použijeme tradiční popis, dostaneme se nevyhnutelně k paradoxním závěrům. Matematická tvrzení odkazující k nekonečnému množství mohou být příkladem těchto tvrzení. V předloženém článku vyložím Dummettův postoj ústící především do manifestačního a akvizičního argumentu, podle nichž by nemělo být vůbec možné nerozhodnutelným tvrzením rozumět. V závěru však ukážu, že jeho záměr – přes mnoho cenných pozorování – selhává, tj. že lze nalézt způsob, jak se oběma argumentům vyhnout a zachovat přitom celistvost realistického popisu jazyka.

Klíčová slova: anti-realismus, matematický výrok, význam Michael Dummett, pravda, pravdivostní podmínka, teorie významu

Štěpánek, J. (2022): „Dummett's Anti-Realism about Mathematical Statements." *Filosofie dnes* 14 (2): 31–49. Dostupné z www.filosofiednes.ff.uhk.cz